Dynamic Path-Based Equilibrium Assignment With Microscopic Traffic Simulation

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DYNAMIC PATH-BASED EQUILIBRIUM ASSIGNMENT WITH MICROSCOPIC TRAFFIC SIMULATION

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EXECUTIVE SUMMARY

This report summarizes research work conducted under TO4158 at the California ATMS Testbed of the Institute of Transportation Studies at the University of California, Irvine. Under this task order, the California ATMS testbed hosted two full-time PATH research postdocs (Henry Liu and Lianyu Chu) whose general responsibilities are focused on applications of ATMS in the Testbed environment. They are generally responsible for ensuring that the functional capabilities of the Testbed are designed, developed and maintained in a manner that complements and enhances the ATMS research objectives of the PATH program. Under the direction of PATH faculty researchers at UCI and the Testbed management team, they are generally responsible for software enhancements to the Laboratory "bench-top" system for modeling and evaluating ATMS, particularly with the microscopic simulation model Paramics. They also provided Caltrans the on-call support and technical guidance on various Caltrans micro-simulation projects related to the Paramics plugin modules developed at UCI.

In addition to the general responsibilities outlined above, the PATH Researchers have specific research projects. The specific research project conducted under this task order is to develop a path-based equilibrium assignment model with micro-simulator Paramics. Since off-the-shlef Paramics can only do the link-based assignment, no path information during the traffic assignment process is stored nor provided. This will bring difficulties in the ATMS evaluation particularly related to the route diversion because partial or full path information is needed to conduct this type of evaluation with Paramics. To overcome this difficulty, a path-based assignment model is developed using Paramics Application Programming Interfaces (API) functions. The model comprises the advantages of the analytical traffic assignment model and the good properties of the simulation tools, which can represented the real world more properly. Some key techniques, route choice plug-in, turn penalty consideration and methods of successive average, and etc., were applied in the model. The Paramics V4 was selected as the demonstration simulation tool. A real grid network in Tucson, Arizona, was designed to test the performance of the model, and the results showed that the model converged to the user equilibrium as expected.

Keywords: Traffic Assignment, User Equilibrium, Traffic Simulation, Turn Penalty, Paramics.

INTRODUCTION

The deployment of intelligent transportation systems (ITS) must be assisted by suitable tools to conduct the feasibility studies and evaluate the expected impacts. In this context, traffic assignment model, which is the last stage of the conventional four-step model in the transportation planning, can be used to help public authorities to test the impact of various transportation policies and ITS deployments. In traffic assignment model, a set of rules and principles towards certain objectives (such as user equilibrium) is used to load a fixed trip matrix onto the network, thus generate a set of link or path flows *(1)*.

There are two categories of methods that arise in the use of traffic assignment model: analytical modeling methods and simulation modeling methods. The analytical-based models derive the optimality conditions from preset driver behavior principle such as utility maximization, and the sensitivity analysis for different scenarios is easier to perform since the procedure is usually less time consuming. Many research works have been done in this area with various functional forms for cost-flow relationships. However, there are still some drawbacks to most of these analytical models. For example, most analytical models assume that the vehicle has zero length, so the spill-back of congestion can not be represented correctly. Moreover, the variety of the vehicle types and stochasticity of the driver behaviors are also represented properly in most of the analytical models. Therefore simulation models have been suggested to overcome these problems.

The simulation-based models can represent the traffic dynamics in more detail and allow studies on the influence of information system techniques in the context of ITS such as testing the different traffic control strategies. The drawbacks of the simulation models are that the number of attributes involved is normally quite large and the properties of the solution remain uncertain. However, in recent years, the improvements in software and hardware technologies and some additional approaches, like distributed simulation, have made possible to simulate microscopically real networks of sensitive size with acceptable speed. The increasing of the simulation speed also insures that multiple-runs technique could be applied to decrease the uncertainty of the final simulation results. Thus simulation modeling becomes an increasingly popular and effective tool for analyzing transportation problems, which are not amendable to study by other means. In the transportation simulation field, there is some general agreement that micro-simulation, i.e., a computational resolution down to the level of individual travelers, is now a viable alternative and may be the only answer to a wide variety of problems.

Some previous investigations had been done in this area. Nagel et al. reported the TRANSIMS computational experiments with large scale dynamic traffic assignments (DTA) in the context of a Dallas scenario in 2000 *(2)*. The authors also looked into the issues of uniqueness, variability, robustness and validation. Regarding uniqueness, despite some cautionary notes from a theoretical point of view, there was no indication of "meta-stable" states for the iterations. The variation of the simulation of a given plan set by just changing the random seed was considerable. The results from three different micro-simulations under the same iteration scenario were shown to test for the robustness

of the results under different implementations. The results were found encouraging, also when comparing to reality and with a traditional assignment result.

In 2002, Barcelo and Casas described a heuristic approach to DTA in which two alternative analytical components are used to determine the path flow rates, one based on a stochastic route choice method, and another one based on an approximation to dynamic user equilibrium conditions, while the network loading was done by a microscopic simulation model, AIMSUN. The case study showed a fairly good agreement between the real system and the simulation model *(3)*. In 2003, Mirchandani proposed an iterated route-based CORSIM simulation model, where drivers' experiences in one period were used as the input to the method of successive average (MSA) algorithm to provide traffic assignment for the next period. To test the validity of the approach, traffic assignment was based on travelers choosing routes that minimize travel times experienced in the simulation, and the network loading was examined for the level of equilibrium in the network. Analysis of the results from the assignment-simulation model showed that routes most used for each OD pair had nearly equal travel times and that the assignment-simulation process tends to converge to the traffic equilibrium as the number of periods increases *(4)*.

The specific research project conducted under this task order is to develop a path-based equilibrium assignment model with micro-simulator Paramics. Since off-the-shlef Paramics can only do the link-based assignment, no path information during the traffic assignment process is stored nor provided. This will bring difficulties in the ATMS evaluation particularly related to the route diversion because partial or full path information is needed to conduct this type of evaluation with Paramics. To overcome this difficulty, a path-based assignment model is developed using Paramics Application Programming Interfaces (API) functions. The model comprises the advantages of the analytical traffic assignment model and the good properties of the simulation tools, which can represented the real world more properly. Some key techniques, route choice plug-in, turn penalty consideration and methods of successive average, and etc., were applied in the model. The Paramics V4 was selected as the demonstration simulation tool. A real grid network in Tucson, Arizona, was designed to test the performance of the model, and the results showed that the model converged to the user equilibrium as expected.

PROPOSED METHODOLOGY

Dynamic User Equilibrium

The traffic assignment model usually attempts to approximate to the equilibrium conditions that described by Wardrop in 1952 *(6)*.

> *"Under equilibrium conditions traffic arranges itself in congested networks such that all used routes between OD pair have equal and minimum costs while all unused routes have greater or equal costs."*

This is usually referred as the Wardrop's first principle or user equilibrium. There are no stochastic effects considered here, i.e. all the drivers have the perfect perceive of

the link cost and same decisions. If the equilibrium is not hold, which means some drivers could select another route to minimize his travel cost. So the user equilibrium can be selected as the criterion to evaluate the proposed hybrid traffic assignment-microscopic simulation model. If exclude the randomness of the simulation, all the vehicle routes between one O-D pair have equal and minimum cost comparing to all other unused routes, then the model maybe appropriate, otherwise, the model will be rejected.

Dynamic user equilibrium (DUE) is to find the dynamic traffic patterns, i.e., the time-varying link flows, link inflow rate and exit flow rate, and then travel time and speed for each link, given the network and time-dependent OD flows, by assuming that each of the motorists is trying to minimize his/her individual travel cost. Analytically, DUE can be formulated as a Variation Inequality model shown in Equation 2.

$$\int_0^T \sum_{rs} \sum_p \eta_p^{rs*}(t) \left[f_p^{rs}(t) - f_p^{rs*}(t) \right] dt \geq 0 \qquad (2)$$

$s.t.$ network constraints

Depends on how to calculate the travel cost, the DUE model can be categorized as instantaneous DUE model and ideal DUE model. The instantaneous DUE model uses the snapshot travel time, while the ideal DUE uses the actual travel time as the cost. The conceptual DUE model can be demonstrated in Figure 1 (7). Initially the time dependent O-D matrices and the network performance functions will be loaded to the network, and in the first stage, the dynamic route choice model (Instantaneous or Ideal) will be applied to determine paths and time-dependent path flows, then in the second stage, the dynamic network loading model, e.g. microscopic simulation in this paper, will load network and get the network measures including link time, path time, and etc. Finally after matching the convergence criteria, this DUE model will stop.
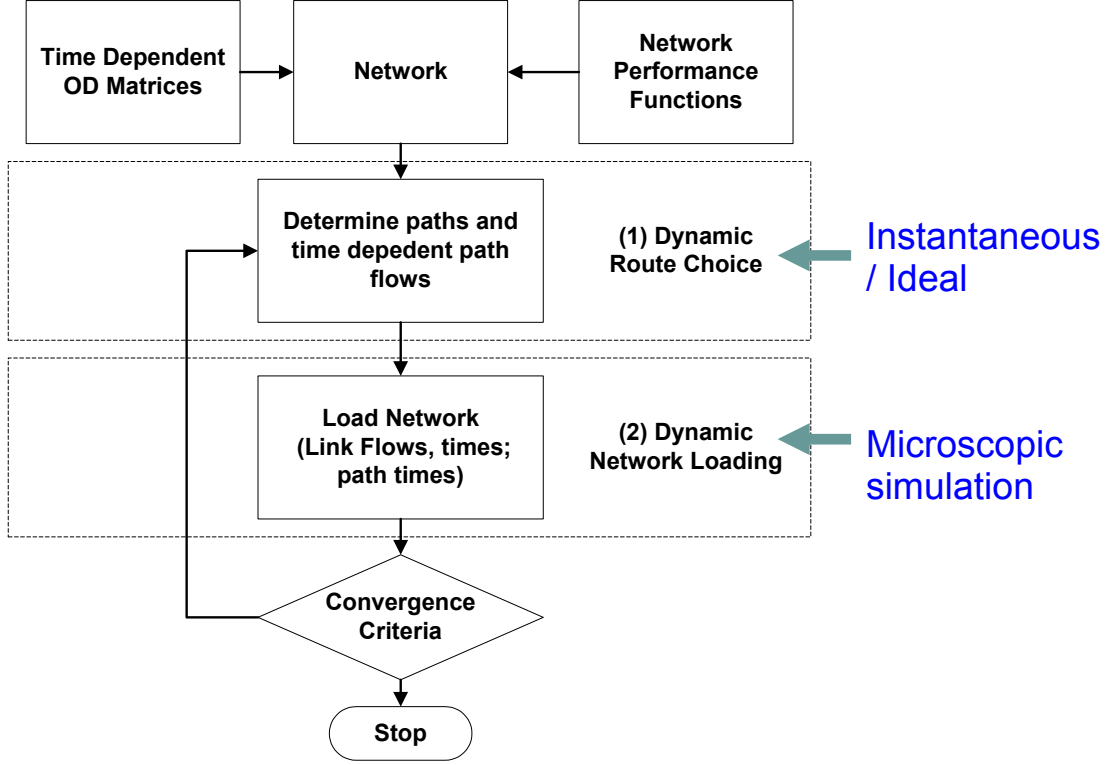
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FIGURE 1   Prototype of the DUE Model.

Simulation-based Traffic Assignment

Essentially, the framework of the DUE model is an iterative route assignment model. In this paper, assume the set of given N user's chosen routes is $\vec{R} = \vec{R}(t) = \{r_1, \dots r_N\}$, the set of L links' costs is $\vec{C} = \vec{C}(t) = \{c_1, \dots c_L\}$, then in the iterated model, there are two transitions as shown below in Equation 3.

$$\text{Simulation}: \quad S: \vec{R} \to \vec{C}$$
$$\text{Assignment}: \quad A: \vec{C} \to \vec{R} \tag{3}$$

In this research, instead of using any specific analytical link cost performance, the microscopic simulator was applied as network loading tool. Comparing with the traditional analytical tools, the embedding of micro-simulator in dynamic traffic assignment 1) could properly or more accurately represent traffic dynamics, such as spill-back of congestion; 2) could properly model network changes, such as work zone and incident; 3) has the capability to incorporate with ITS strategies, such as accurate signal, ramp metering, and variable message signs; and 4) finally micro-simulators are commercially available and professionally supported.

The Frank-Wolfe Algorithm vs. Method of Successive Average

The Frank-Wolfe Algorithm, also known as the Convex Combination Algorithm, was originally proposed by Frank and Wolfe in 1956 as a procedure for solving quadratic programming problems with linear constraints. The method is a feasible direction method, and especially useful for determining the equilibrium flows for transportation networks *(8)*. The Frank-Wolfe is the popular approaches used to solve the traffic assignment problem.

An important step of the Frank-Wolfe algorithm is line search, which is directly determining the speed of the convergence. For analytical traffic assignment models, some differential methods, like Golden Section method or Bisection method, could be applied to obtain good convergence properties. However, for simulation-based traffic assignment models, the link cost is a real value that will be obtained directly from the simulation, thus it is not differentiable. So the Methods of Successive Averages (MSA) is proposed for solving the user equilibrium problems. MSA use the line search step as 1/n, where n is the number of iterations. It can be approved that the convergence of MSA is non-monotonic and guaranteed.

Another key technique in traffic assignment model is the shortest path algorithm. Dijkstra's algorithm *(9)* is one of the most efficient and simple shortest path algorithms. Dijkstra's algorithm is designed to find the lowest cost path between a single starting node and all of the other nodes in a graph. It can also be used to obtain the lowest-cost path between all nodes if it is called repeatedly for every node in a graph. Dijkstra's algorithm is a greedy algorithm which means, if given a choice, it operates by choosing the highest or most valuable alternative.

In this research, we will embed MSA traffic assignment as an important component of the proposed hybrid model. Instead of using the cost-flow functions, like BPR function *(10)*, to estimate the link cost, the microscopic simulation will be applied.

Convergence Criteria

The relative gap function shown as Equation 4 is selected as the convergence condition in this model.

$$R_{gap}(t) = \frac{\sum\limits_{\forall r,s} \sum\limits_{p \in P^{rs}(t)} f_p^{rs}(t) \cdot [\eta_p^{rs}(t) - \pi^{rs}(t)]}{\sum\limits_{\forall r,s} d^{rs}(t) \cdot \pi^{rs}(t)} \tag{4}$$

Where $f_p^{rs}(t)$ is the flow entered network at time interval *t* between O-D pair *r-s* via path *p*; $\eta_p^{rs}(t)$ and $\pi^{rs}(t)$ are the actual and minimum travel cost on path *p* at time interval *t* to flow $f_p^{rs}(t)$ respectively; $d^{rs}(t)$ is the demand departing from origin during t^{th} time interval between O-D pair *r-s,*; and $P^{rs}(t)$ is the path set between O-D pair *r-s* at time interval *t*. If the R_{gap} approaches to zero, then perfect equilibrium condition will be

obtained. However, it is hard to get such equilibrium condition in reality, so the model is try to reduce the R_{gap} to certain predefined criteria.

Flowchart

Figure 2 shows the flowchart of the proposed hybrid traffic assignment-microscopic simulation model. First the model will load the network information, O-D matrix demand, and the initial link costs which are the free flow travel time pre-generated by Paramics. Initialize all the flow $V_a = 0$, and the iteration number n = 0. Next use the Dijkstra's algorithm to find the shortest paths of all O-D pairs with the current link cost information that generated from the Paramics simulator, and increase the number of iterations by 1. Then make an All-or-nothing assignment to obtain the auxiliary flow F_a, for all link a, Set link flow $V_a^n = (1-\varphi)V_a^{n-1} + \varphi F_a^n$. According to the good property of MSA, the current updated link flow is the average of the auxiliary flow of all previous iterations. Send the flow information to Paramics simulator and execute a new iterative simulation with route choice mode and turn penalty associated link cost model. Run this iterative hybrid model until the convergence criterion arrived, which is guaranteed by MSA.
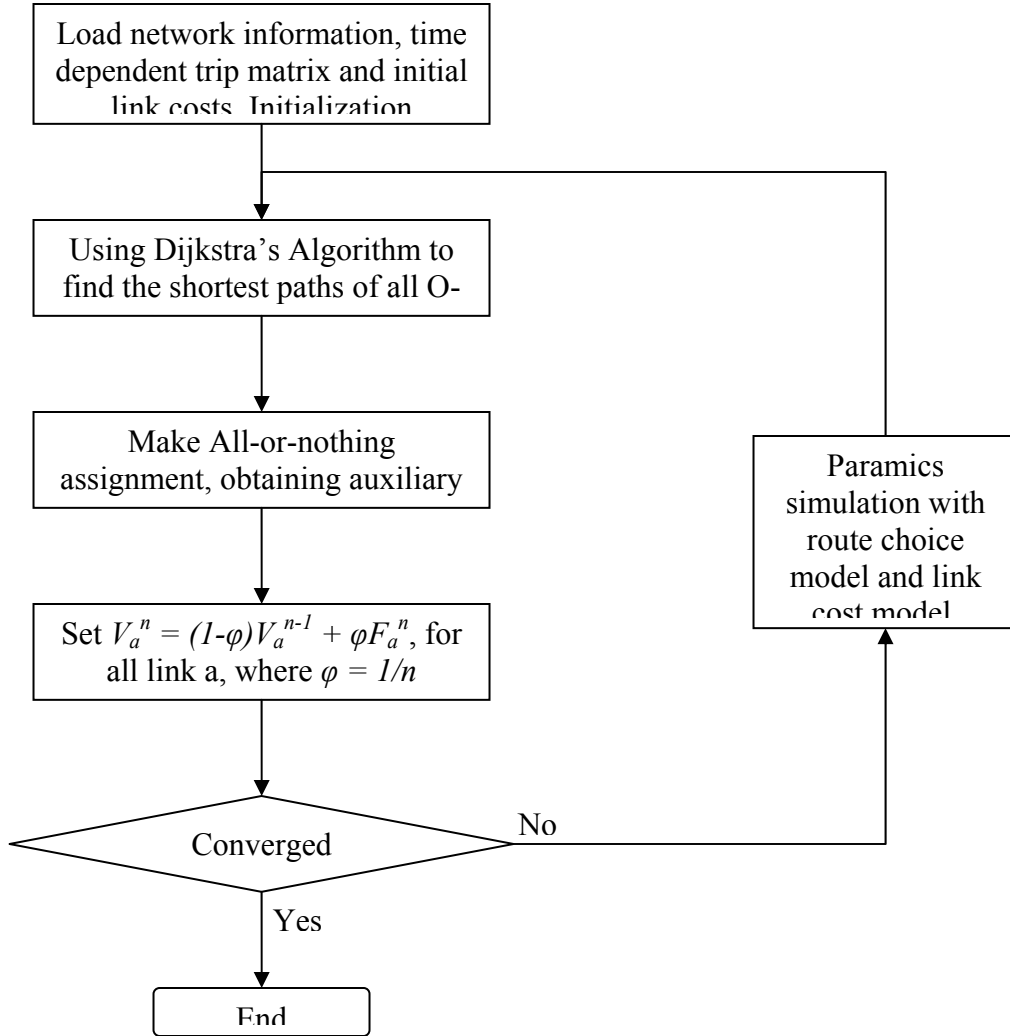
FIGURE 2   Flowchart of the Proposed Model.

MODEL IMPLEMENTATION

Paramics Simulation Tools

Paramics is a suite of microscopic simulation tools used to model the movement and behavior of individual vehicles on urban and highway road networks *(11)*. It simulates the components of traffic flow and congestion, and presents its graphical animation output simultaneously for traffic management and road network design. One important feature of Paramics is that it allows the user to customize many features of underlying simulation model through API. In general, the API functions can be used to override and extend the default functionalities or logic, and obtain and set the simulation parameters from Paramics. Through the API, relevant link cost functions are programmed to interface with Paramics. In addition, users can replace the default route choice model in

Paramics by using APIs, hence the path-based microscopic simulation model could be established to replace the default link-based model. Comparing with other traffic simulators, the Paramics Version 4 has undergone a significant overhaul. The V4 API is more structured, easier to use, and inherently safer. The breadth of V4 API has also been extended. So Paramics V4 is selected to test the proposed methodology in this paper.

Route Choice Model

The evaluation of the proposed traffic assignment model requires a mechanism to simulate traffic loads on user-defined paths. Currently, most of the commercially available microscopic simulation models do not have an option to load traffic on paths. However, the Paramics API support the user to build their own route choice model to simulate the vehicles with specific routes on a turn by turn basis. First, the DUE model will generate the paths, which will be received by the route assignment plug-in. Every time there is a new vehicle released, the route assignment plug-in will automatically associate a network path to this vehicle, and its movement or link transfer will be determined by the associated path. So combined with other plug-ins, such as link performance data collection, the Paramics will collect the link cost and feedback to the DUE model.

Turn Penalty

Another issue arises in the Paramics simulation is the associated turn penalty of the link cost. At the intersection or junction of the urban network, different turn movements have different travel cost. For example, the un-protected left turns at a signalized intersection usually need to yield to the through movements, then left turn vehicles will have high associate cost than the through vehicles. This additional cost that associated with the movement is so-called turn penalties. In this paper, we define the turn penalty as the actual link travel time minus the link free-flow-travel-time. In Paramics, the turn penalties are associated with the downstream links, which will cause problems when applying the traditional shortest path algorithms, like Dijkstra's. Because in every loop, the minimum cost from the origin node to the current labeled node is not a fixed value due to the uncertainty of the next turn. To solve this problem, we introduce a dummy node in each link, and the link cost of each link consist of two parts: free-flow-travel-time (the upstream node to the dummy node), and turn penalty (dummy node to the downstream node). Then we redefine the link cost as the summation of the free-flow-travel-time and the turn penalty of the processor link instead of the current link. Since every time we know the route from origin node to current node, so all link costs are known by this redefinition. Note that one link may have three or more costs due to the association of the different injection link turn penalty, in the Dijkstra's algorithm, an additional judgment of previous node was used to determine which value will be selected as the current link cost.

COMPUTATIONAL RESULTS

An example network of the University of Arizona campus area at Tucson, Arizona, was built to test the performance of the proposed hybrid traffic assignment-microscopic

simulation model. The network shown in Figure 3 is a well calibrated gird network. It is coded by 29 O-D demand zones, 21 actuated signalized intersection in Paramics Modeler Version 4.21. The east-west direction is about 4 miles, and the north-south direction is about 2 miles. Two peak-hours demand was assigned in the network, and the demand pattern is shown as Figure 4. Testing was carried on a desktop with one 2.8 GHz CPU processor and 1 GB RAM. The operation systems are Windows XP Professional.
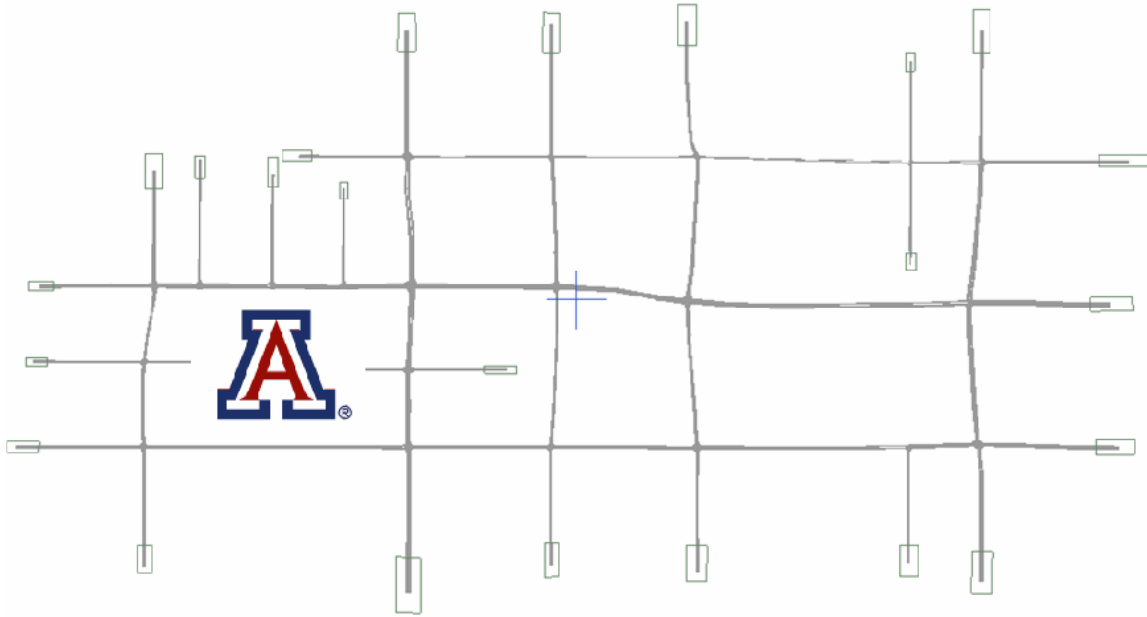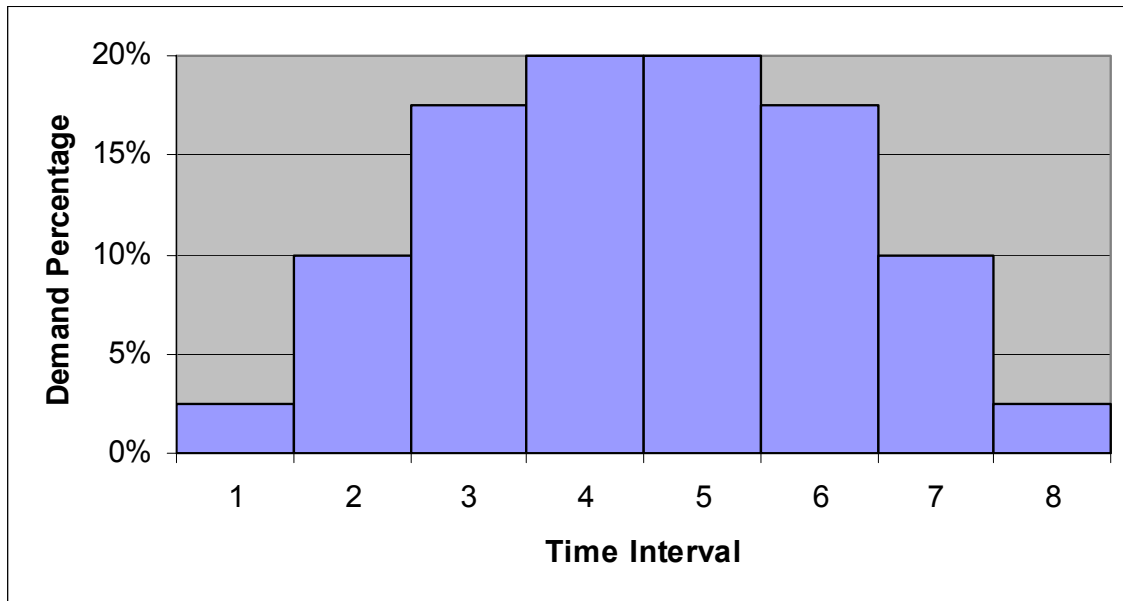

FIGURE 3   Tucson Grid Network.

FIGURE 4   Demand Pattern.

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The total simulation time is 2 hours, and each time interval in the DUE model is equal to 1 minute, so there are total 120 time intervals. The convergence criteria used in this scenario is $R_{gap} = 10^{-4}$ for all time intervals, and the maximum iteration is 50. Although both instantaneous and ideal DUE models were implemented, we will only shown the ideal DUE case here for there are not a big difference in this example network.

Figure 5 shows the convergence curve of R_{gap} value after 50 iterations. It is obviously that the relative gap drops very quickly in the first several iterations, from around 1.1% to near 0.1%, and after that the R_{gap} start to oscillate. This oscillation is largely due to the stochastic nature of the microscopic simulation model. A specific O-D pair from zone 5 to zone 27 was selected to analyze, we found there are totally 6 different routes were used to assign the demand, and these routes avoid the congested links or intersections. Figure 6 shows the route travel time pattern from zone 5 to zone 27 over the whole study period. We can see that it has the similar pattern as demand pattern, and the used paths only have around 5% difference, which means the 6 assigned routes are reasonable. Figure 7(a) shows the improvement in link travel time at equilibrium conditions. The link (link number 24:11) is congested after 30 minutes of simulation in the initial iteration, while it is quite smooth and no congestion occurred after 50 iterations when the equilibrium condition obtained. Figure 7(b) shows the detailed scale pattern of the link travel time, and it is also similar as the demand pattern.
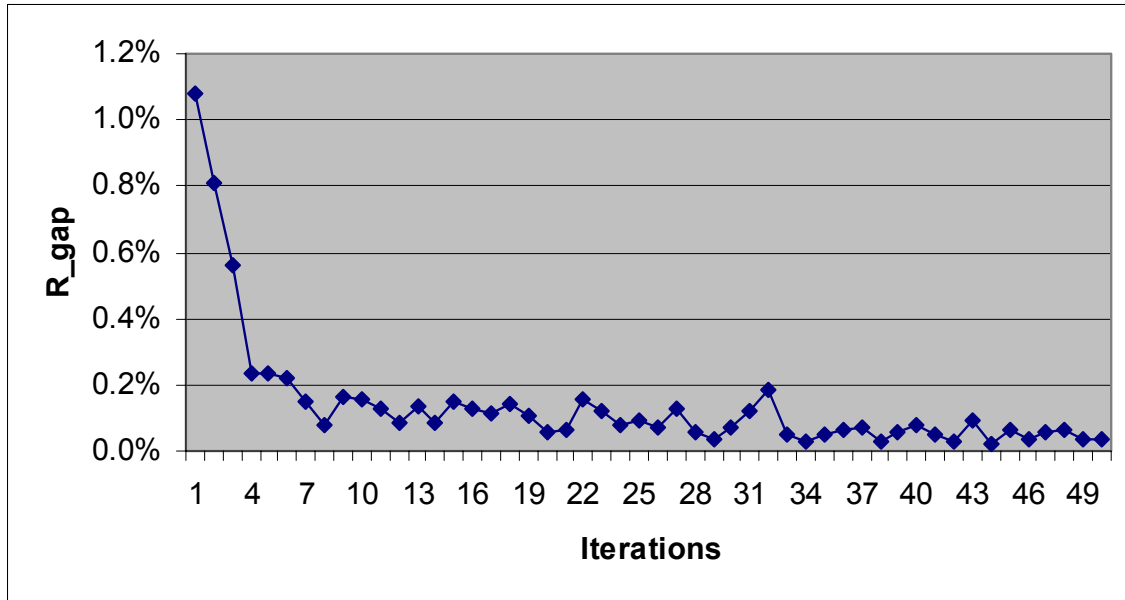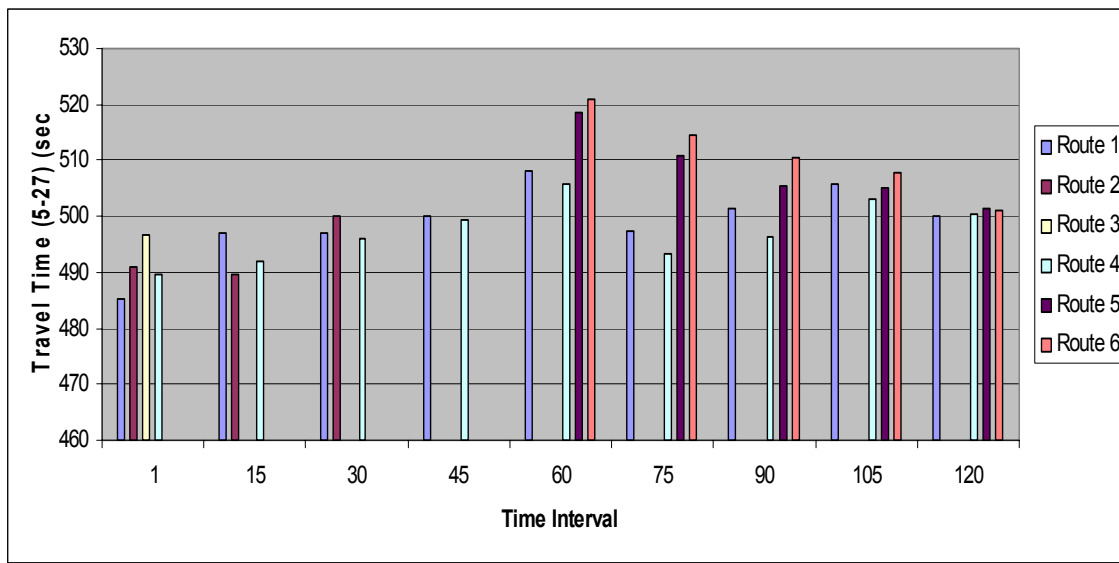

FIGURE 5 R_{gap} Convergence Curve.

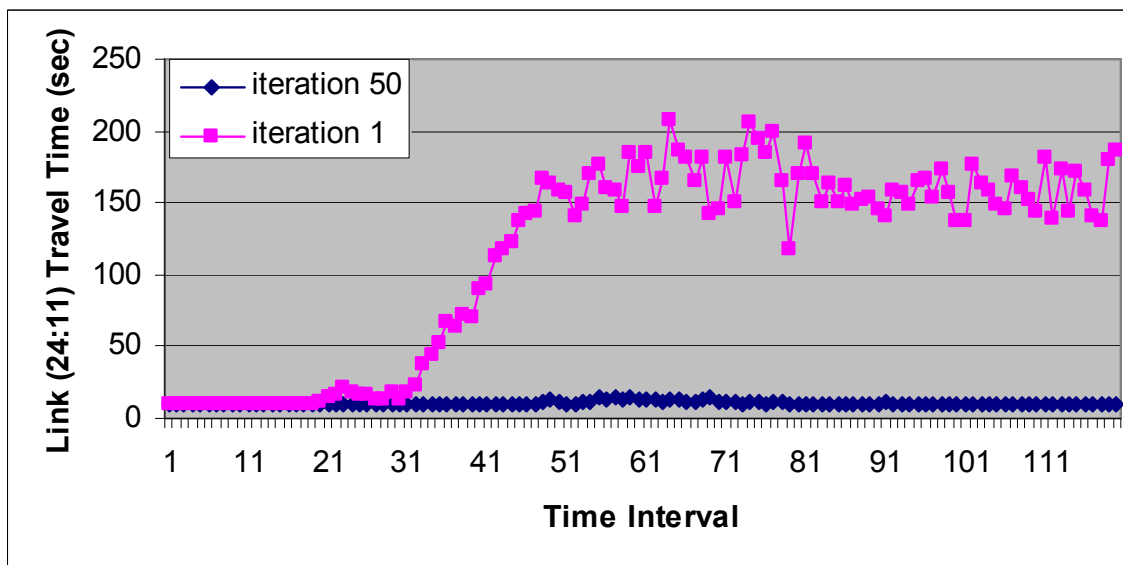FIGURE 6   Route Travel Time Pattern.

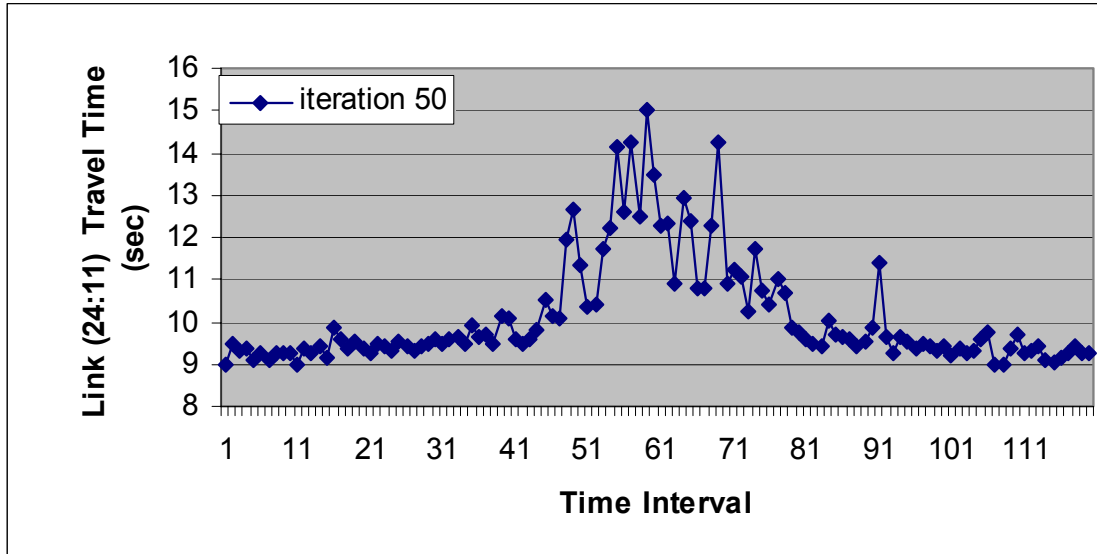FIGURE 7(a) Link Travel Time Improvement.

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FIGURE 7(b) Link Travel Time Pattern.

CONCLUSION AND FUTURE RESEARCH

A dynamic route assignment model with embedding microscopic simulation model is proposed in this paper. The model is comprised by two parts: the analytical traffic assignment model and the simulation demonstration tool. The methods of successive average was used in the analytical traffic assignment model with the embedded microscopic traffic simulator. The popular Dijkstra's algorithm was used in this model. However, the model also considered the turn penalty that associated with the link cost in the shortest path searching. The microscopic simulation software, Paramics V4, was used as the simulator to get the link cost or travel time, instead of the analytical cost-flow functions, most of which could not represent the relationship when the network is jammed or congested. A real grid network at Tucson, Arizona, was coded to test the performance of the proposed model, and the results show that all the route selected have nearly equal and minimum cost and the model will arrive the expected user equilibrium after several iterations.

For the further research, besides apply better assignment approach for the time dependent path flow rates, the authors also want to replace MSA with more efficient line search method, and apply the model with the Advanced Traffic Management System (ATMS) applications.

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